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APPLICATION OF MULTIOBJECTIVE OPTIMIZATION TOOLS IN A THRUST BEARING PAD MODELLING

Summary: To model an oil film of a fluid friction slide bearing, it usually requires solving a set of equations. Some part of these equations are partial differential equations. Thanks to application of numerical methods, it is possible to find an approximated solution. Most commonly, iterative methods implementing nested loops are suggested, where each loop provides solution for one parameter at fixed values of the remaining parameters. The paper presents an attempt to use mathematical optimisation terms in defining a problem of modelling of an oil film created in the face of a hydrostatic thrust bearing pad. Multiobjective optimisation tools have been applied. Some parameters of the oil film were treated as unknowns, while the others as the goal functions. One of the advantages of such approach may be reducing the number of iterations and speeding up the computing process.
Keywords: Thrust bearing, hydrostatic oil film, multiobjective optimisation.

INTRODUCTION

The phenomena occurring in the oil film of the fluid friction slide bearings are described with partial differential equations. Depending on the complexity of the assumed model, the number of the equations may vary. These include the Reynolds equation, energy equation, interspace elastic deformation equations etc. Additionally, the model is completed with another equations (oil viscosity function on the temperature, oil film thickness function etc.) Even the simplest, isothermal model includes a set of equations, whose analytic solution, if not impossible, is very difficult. Nowadays, numerical methods, which allow finding an approximate description of the system's state, are ubiquitous. In particular, iterative methods have found vast application. In such methods, the error of the solution is reduced in every step using different algorithms, until the desired precision is achieved.

In many works [3, 8, 9], nested loops are suggested (Fig. 1). Specific equations, being the part of the model are solved using specific methods, and their results become inputs for the subsequent ones. If, in a given stage of solution in a given iteration, the difference between initial and final value of a specific variable is greater than the assumed tolerance, some parameters are modified, and the procedure starts all over. Such design of an algorithm may cause large number of iterations, and therefore, increase the resources requirements.

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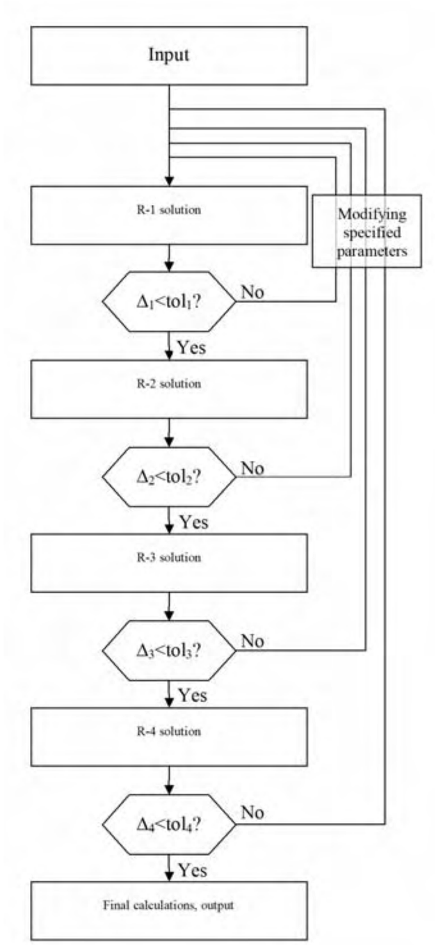

Fig. 1. Simplified scheme of the iterative procedures used for determining the oil film parameters. R-1, R-2,… - subsequent equations and functions dependant on specified parameters; Δ_i – difference between the initial and final value of a specified variable in the beginning and end of a loop; tol_i – tolerances

Rys. 1. Uproszczony schemat stosowanych procedur iteracyjnych określających parametry filmu olejowego. R-1, R-2,… - kolejne równania i funkcje zależne od określonych parametrów; Δ_i – różnica między daną wielkością na początku i końcu danej iteracji; tol_i – tolerancje

In many fields of science, operative research and optimization theory have found application. In mechanical engineering, these tools are commonly used for optimization of a construction or process design, with regard to different, often contradictory criteria (eg. maximizing the strength of a structure and minimizing its weight) [4, 5]. From the point of view of the mathematical theory of optimization, the solution of a problem reduces to finding a minimum (or maximum) of a function or a vector of functions, called the objective functions, considering given constraints. The variables of these functions are called parameters or unknowns [6]. If the objective functions, constraints and the parameters are properly defined, the optimization tools may be applied for modelling a state of an oil film.

OIL FILM MODEL

The tools of multiobjective optimization have been applied in modelling an oil film hydrostatically generated on a surface of a bearing pad of a thrust bearing of one of the Polish Hydro Power Stations (Fig. 2). To ensure achieving the hydrodynamic lubrication conditions at nominal revolution speeds, the bearing pads are rested on sets of helical springs. The springs parameters (spring rate, and length) are significantly dispersed, which is a consequence of difficulties in manufacturing. In the past, an improper selection of springs parameters has led to a series of failures at the hydrostatic phase of the machines operation [2]. Some modeling has been performed to determine the probability, that due to improperly selected springs, a metallic contact of the bearing’s mating faces occurs, leading to a loss of fluid friction at the initial phase of the bearing’s operation.

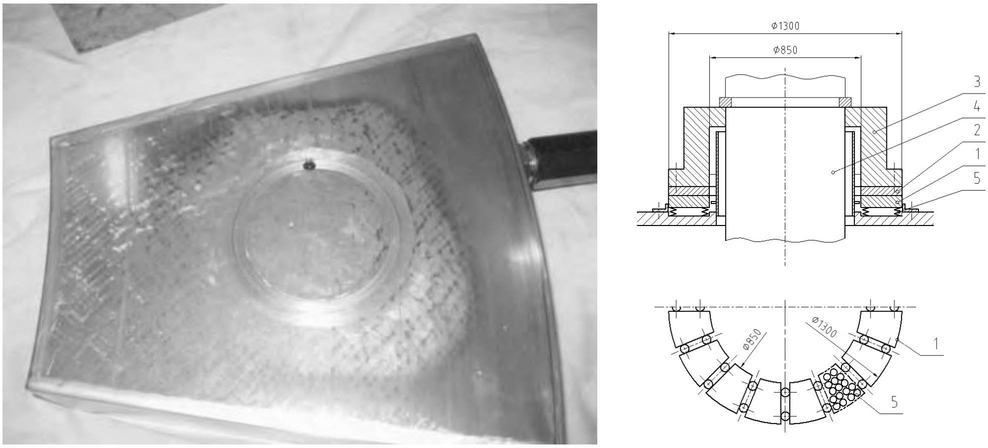

Fig. 2. Hydro generator thrust bearing. a) photo – excessive surface wear due to metallic contact (failure) visible; b) spring support scheme: 1 – bearing pad, 2 – slide ring, 3 – flanged bushing, 4 – generator shaft, 5 – set of supporting springs
Rys. 2. Łożysko wzdłużne hydrozespołu: a) fotografia – widoczne zużycie powierzchni wywołane awarią; b) schemat podparcia sprężystego: 1 – segment łożyska, 2 – pierścień ślizgowy, 3 – tuleja kołnierzowa, 4 – wał generatora, 5 – zespół sprężyn podpierających

A part of the modelling was to determine the oil film parameters in case of direct metallic contact of the mating faces in a specified point (Fig. 3). Due to low number of parameters, this part of modelling will further exemplify the idea of using optimization tools in modelling of an oil film.

The experimental measurement of pressure distribution in an inclined interspace [1] has shown, that that the isothermal oil film model (that is at assumed constant oil viscosity and non-deformable interspace faces) describes the pressure distribution in the hydrostatic, initial phase of bearing operation with satisfying precision. Thus the isothermal model has been used.

In order to model the oil film, the following assumptions have been made:

– The oil flow is laminar, and the oil is incompressible
– The surfaces defining the interspace are non-deformable. This allows simple definition of the oil film thickness as $h(x,y)=0-(A\cdot x+B\cdot y+C)=-A\cdot x-B\cdot y-C$ (Fig. 3)
– The following have constant values: oil viscosity (isothermal model) η = 3,496·10^{-8} [MPa·s], oil feed from the oil groove (positive displacement pump applied) $\dot{Q}=533333\left[\mathrm{mm}^3/\mathrm{s}\right]$, bearing pad load G = 134 375 [N].

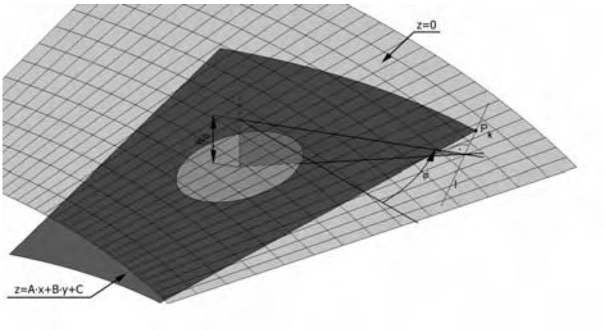

Fig. 3. Geometrical dependencies in case of metallic contact of the bearing pad surface and the slide ring surface: $z = A \cdot x + B \cdot y + C$ - bearing pad surface; $z = 0$ - slide ring surface. φ – the direction ("azimuth") of the bearing pad surface inclination; P_k – contact point; l – the line of the two planes common points; h_0 – oil film thickness in the middle of the oil groove

Rys. 3. Ilustracja zależności geometrycznych w przypadku metalicznego kontaktu współpracujących powierzchni ślizgowych. $z = A \cdot x + B \cdot y + C$ - powierzchnia segmentu; $z = 0$ - powierzchnia pierścienia ślizgowego. φ - kierunek nachylenia płaszczyzny segmentu; P_k – punkt styku; l – prosta na przecięciu obydwu płaszczyzn; h_0 – grubość filmu olejowego w środku komory smarowej

The pressure distribution in the interspace can be described with Reynolds equation:

$$\nabla\left(-\frac{h^3}{12\eta}\cdot\nabla p\right) = -\frac{1}{2}\nabla\left(\vec{V}\cdot h\right) \tag{1}$$

where:
p – pressure [MPa],
h – oil film thickness [mm],
η – oil dynamic viscosity [MPa·s],
V – relative linear speed of the slide ring on the bearing pad [mm/s].

The approximate solution of equation (1) was determined with the use of Finite Elements Method implemented in MatLab environment (Fig. 4).

Determining the pressure distribution allows determining the value of the total pressure force (load capacity) of the pad:

$$G = \int_A p(x, y)\, dA \tag{2}$$

where: *dA* – slide surface elementary section, as well as determining the oil flow from the oil groove:

$$\dot{Q} = -\frac{1}{12\cdot\eta}\cdot R\int_0^{2\pi} h(\varphi)^3\cdot\vec{\nabla}p(\varphi)\cdot\vec{n}\, d\varphi + \frac{1}{2}R\int_0^{2\pi} h(\varphi)\cdot\vec{V}(\varphi) \tag{3}$$

where *R* – radius of the oil groove [mm].

In case of the metallic contact in the P_k point, additionally, the coefficients A, B, and C are bound with geometrical dependencies (Fig. 3): $A/B = \tan(\varphi)$, $h_{\min} = 0$ (in the P_k point).

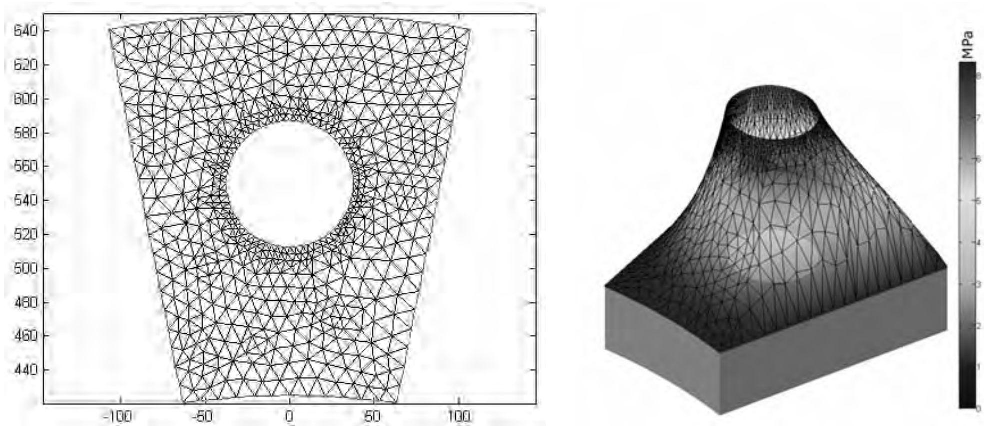

Fig. 4. Numerical solution of Reynolds Equation (1): a) node mesh, approximating the bearing pad surface for FEM application; b) example of pressure distribution in the hydrostatic oil film on the bearing pad surface, determined with FEM

Rys. 4. Numeryczne rozwiązywanie równania Reynoldsa (1): a) siatka punktów, przybliżająca powierzchnię segmentu do zastosowania MES; b) przykładowy rozkład

In such case, the problem may be reduced to finding two parameters: p_0 and h_0. According to the third Newton's law, the equation (2) should yield exactly the value of load *G*, and according to the law of mass preservation, the result of equation (3) should be equal to the value of feed $\dot{Q}$.

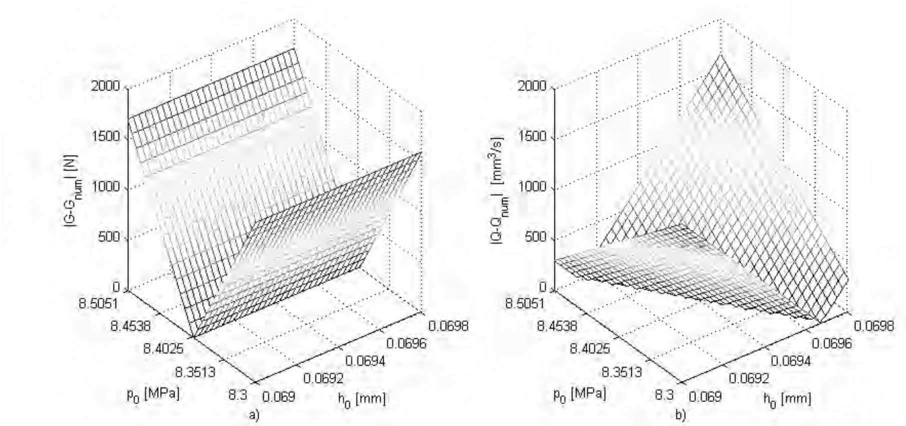

Fig. 5. Dependency of the absolute error of computing a) of the pressure force G_{num} and b) oil flux from the oil groove $\dot{Q}_{num}$ on the parameters p_0 and h_0. Angle of the bearing pad inclination: $\varphi = 60°$

Rys. 5. Zależność absolutnego błędu obliczenia a) siły naporu G_{num}, oraz b) natężenia wypływu oleju z komory smarowej Q_{num} w zależności od parametrów decyzyjnych. Kierunek pochylenia powierzchni segmentu: $\varphi = 60°$

At the revolution speed equal zero, the equation (1) is influenced only by the A, B and C coefficients, describing the oil film thickness *h*, so – in the discussed case of metallic contact – (1) depends indirectly on h_0, and on the oil groove oil pressure p_0 (boundary condition for the Reynolds Equation). Determining the pressure distribution is therefore conditioned by determining these two parameters.

Fig. 5 presents, how the absolute value between the given bearing load, *G*, and the total pressure force G_{num} computed by numerical integration of pressure distribution, and between the given pump feed $\dot{Q}$ and numerically integrated (3) flux $\dot{Q}_{num}$ from the oil groove, depend on the values of p_0 and h_0. It is clear, that for both differences, for some values of p_0 and h_0 they achieve their minima. It can be said, that for any value of h_0 and p_0, we get a certain set of equations, which describe the oil film. Due to the numerical character of the computations, one has to accept certain errors, like the ones resulting from FEM approximation of the oil film distribution function, or the consequent pressure gradient errors in (3). The question arises, for which values of h_0 and p_0, the set of equations will reflect the third Newton's law, and the law of mass preservation with the greatest possible precision:

$$\begin{cases} G_{num} = G \\ \dot{Q}_{num} = \dot{Q} \end{cases} \qquad \begin{cases} \left| G_{num} - G \right| = 0 \\ \left| \dot{Q}_{num} - \dot{Q} \right| = 0 \end{cases} \tag{4}$$

Therefore we intend to minimize a set of objective functions (4). In most common applications of the optimization procedures, the objective functions are mutually contradictory. In some cases though, a so called ideal solution exists [7], when from all the available solutions one can select the one for which all the objective functions can be extremized. For the physical processes, like formation of a hydrostatic oil film and definition of the bearing pad inclination, it can be assumed, that as long as the equations describing the modelled process do not defy the laws of physics, the set of equations should have its ideal solution.

Nowadays a vast range of mathematical optimization is available. In the discussed case, an Optimization Toolbox™ 5 in MatLab has been used. Particularly, the `fgoalattain` function has been utilized, which iteratively solves multiobjective goal attainment problems , using sequential quadratic programming [10]. Rather than changing one variable at a time, like in the nested loops algorithms, the `fgoalattain` algorithm concerns the minimization of a set of objectives simultaneously. The formal notation of the task being executed is:

$$\text{Minimize } \max\left(\frac{F_i(x) - F_i^*}{w_i} \right) \tag{5}$$

where:

F_i – i-th element of the objective function $F = \begin{bmatrix} G_{num}(x) \\ \dot{Q}_{num}(x) \end{bmatrix}$,

F_i^* – i-th element of the goal vector $F^* = \begin{bmatrix} G \\ \dot{Q} \end{bmatrix}$, x – function parameters vector $x = \begin{bmatrix} h_0 \\ p_0 \end{bmatrix}$, w_i – weighing factors vector $w = \begin{bmatrix} G \\ \dot{Q} \end{bmatrix}$.

Setting the weighing factors vector w equal to F^* makes the goal attainment problem become minimizing the relative difference between the functions $F_i(x)$ and the goals F_i^*. Additionally, the following constraints have been defined:

$$h_0 \in \langle 0 \quad 0{,}05mm \rangle, \; p_0 \in \left\langle \frac{G}{F_p} \quad \frac{G}{F_g} \right\rangle \tag{6}$$

where: F_p – bearing pad area; F_g – oil groove area.

OPTIMIZATION RESULTS

Fig. 6 presents the best fitted values of h_0 and p_0 parameters for different directions ("azimuths") of bearing pad inclination φ, at 1° increment of φ. Fig. 7 presents the relative error of the goal attainment, defined as:

$$\frac{\left|G - G_{num}\right|}{G}, \; \frac{\left|\dot{Q} - \dot{Q}_{num}\right|}{\dot{Q}} \tag{7}$$

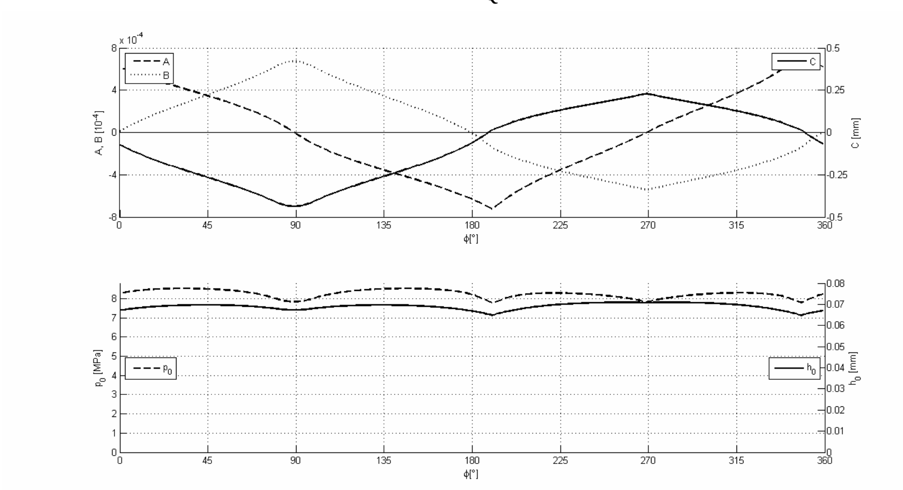

Fig. 6. Values of h_0 and p_0 determined for different directions of bearing pad inclination φ in case of metallic contact of the mating faces, and corresponding values of the A, B, and C coefficients, describing the plane of the bearing pad face.

Rys. 6. Określone wartości h_0 i p_0 dla różnych kierunków pochylenia segmentu φ w przypadku kontaktu współpracujących powierzchni, oraz odpowiadające tym kątom wartości współczynników A, B, i C, opisujących płaszczyznę powierzchni ślizgowej segmentu

The average time of solution was 8,57s. Small values of errors (7) are worth noting (Fig. 7).

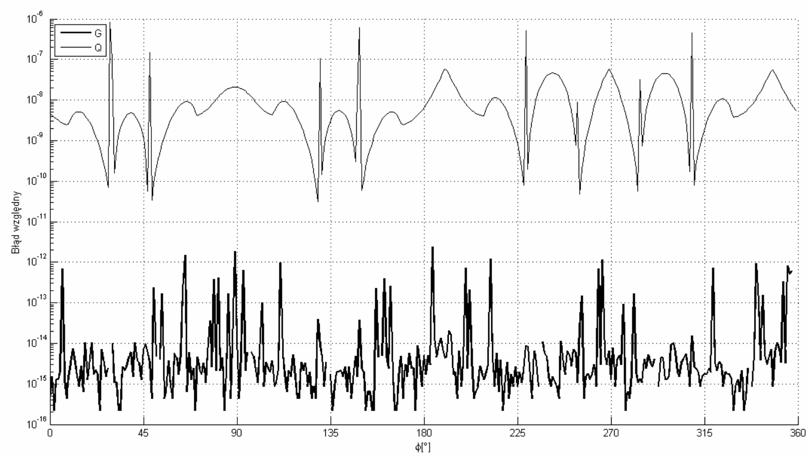

Fig. 7. Relative error of objective function goal attainment according to (7)
Rys. 7. Względny błąd określenia funkcji celu według równań (7)

DEVELOPMENT

One can expect, that the multiobjective optimization tools can be applied in problems with the number of unknowns greater than two. In the above mentioned process of bearing pad oil film modelling, the assumption of the metallic contact has been suspended, and the spring support was considered. The vectors in equation (6) became longer:

$$x = \begin{bmatrix} A \\ B \\ C \\ p_0 \end{bmatrix} \qquad F = \begin{bmatrix} G \\ \dot{Q} \\ 0 \\ 0 \end{bmatrix} \quad F^* = \begin{bmatrix} G_{num}(x) \\ \dot{Q}_{num}(x) \\ x_{oil} - x_{spr} \\ y_{oil} - y_{spr} \end{bmatrix} \tag{8}$$

where:

- x_{oil}, y_{oil} – coordinates of oil pressure force application point F_{oil} (Fig. 8),
- x_{spr}, y_{spr} – coordinates of the point of application of spring support resultant force F_{spr},
- x_{oil}, y_{oil} coordinates may be determined basing on the pressure distribution function, while x_{spr}, y_{spr} can be calculated basing on springs deflection.

Multiobjective optimization tools were used in modelling of around 5000 cases of the bearing pad supported on randomly generated sets of springs. Single solution, that is determining the A, B, and C and p_0 values for given set of springs lasted for about 120 seconds.

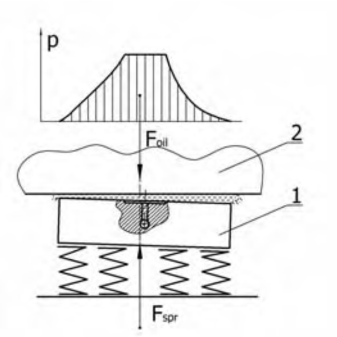

Fig. 8. System of forces acting on the bearing pad during the hydrostatic phase of operation: 1 – bearing pad; 2 – slide ring; F_{spr} I resultant force of spring support; F_{oil} – total pressure force; p – pressure distribution

Rys. 8. Układ sił działających na segment podczas smarowania hydrostatycznego: 1 – segment łożyska; 2 – pierścień ślizgowy; F_{spr} – wypadkowa siła podparcia sprężynami; F_{oil} – wypadkowa siła naporu oleju na powierzchnię segmentu; p – ciśnienie filmu olejowego

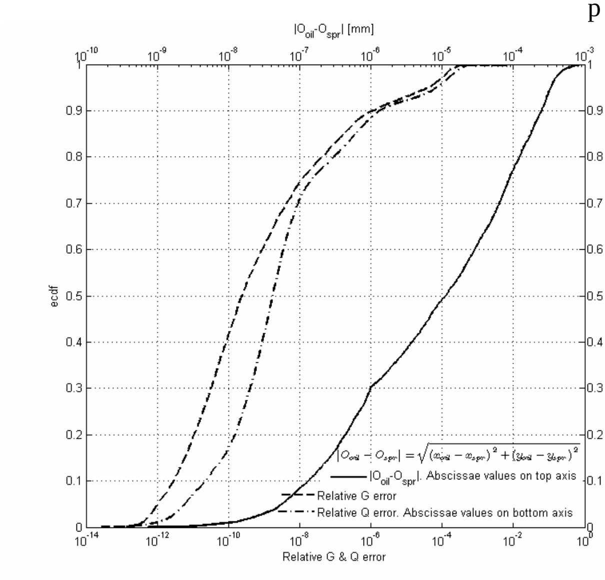

Fig. 9. Empirical Cumulative Distribution Functions (ecdf) of the relative errors (7) and the absolute distance between the points of application of the F_{oil} and F_{spr} forces for the simulations performed

Rys. 9. Dystrybuanty względnych błędów (7), oraz bezwzględnej różnicy położenia punktów przyłożenia sił dla przeprowadzonych symulacji.

Fig. 9 presents the Empirical Cumulative Distribution Functions (ecdf) for the relative errors (8) of G and $\dot{Q}$, and the absolute distances between the points of application of the forces acting on the bearing pad, observed for the above mentioned 5000 simulations. It is worth noting, that for about 90% of cases, the relative errors (8) were not greater than 10^{-6}, and the co-linearity of the acting forces was determined with precision better than 1μm. Application of numerical methods for solving (1) and (3) themselves is expected to yield greater errors.

SUMMARY

The examples above show, that after proper definition of the problem, the mathematical optimization tools can be used in modelling physical phenomena. As long as the state of the system can be described with a set of equations, in the terms of optimization an ideal solution can be expected to exist. Using the optimization tools allows determining the unknown parameters of such system at smaller cost and resources consumption, than the iterative procedures with nested loops, described in literature. The tools and software available in the market allows free definition of problems as well as setting the desired precision of solution.

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ZASTOSOWANIE NARZĘDZI OPTYMALIZACJI WIELOKRYTERIALNEJ DO MODELOWANIA HYDROSTATYCZNYCH ŁOŻYSK WZDŁUŻNYCH

Streszczenie

Modelowanie filmu olejowego ślizgowego łożyska tarcia płynnego wymaga najczęściej rozwiązania układu kilku równań, z czego część to równania różniczkowe cząstkowe. Dzięki zastosowaniu metod numerycznych możliwe jest znalezienie przybliżonego rozwiązania takiego układu. Najczęściej proponowane są metody iteracyjne z zagnieżdżonymi pętlami, z których każda określa rozwiązanie jednego równania przy ustalonych parametrach innych. W pracy przedstawiono próbę zdefiniowania zagadnienia modelowania filmu olejowego na powierzchni segmentu hydrostatycznego łożyska wzdłużnego pojęciami optymalizacji matematycznej. Zastosowano narzędzia optymalizacji wielokryterialnej. Część parametrów filmu olejowego potraktowano jako zmienne decyzyjne, inne natomiast jako funkcje celu. Korzyścią płynącą z takiego podejścia może być zmniejszenie liczby iteracji i przyspieszenie obliczeń.

Słowa kluczowe: Łożysko wzdłużne, hydrostatyczny film olejowy, optymalizacja wielokryterialna.